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$VA\widetilde{V}$ correlator within the instanton vacuum model

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Abstract. The correlator of vector and non-singlet axial-vector currents in the external electromagnetic field is calculated within the instanton liquid model of the QCD vacuum. In general the correlator has two Lorentz structures: longitudinal, $w_{\rm L}$, and transversal, $w_{\rm T}$, with respect to the axial-vector index. Within the instanton model the saturation of the anomalous $w_{\rm L}$ structure is demonstrated. It is known that in the chiral limit the transversal structure $w_{\rm T}$ is free from perturbative corrections. In this limit within the instanton model we calculate the transversal invariant function $w_{\rm T}$ at arbitrary space-like momentum transfer q and show the absence of power corrections to this structure at large q^2. Instead there arise exponential corrections to $w_{\rm T}$ at large q^2 reflecting non-local properties of the QCD vacuum. The slope of $w_{\rm T}$ at zero virtuality, the QCD vacuum magnetic susceptibility of the quark condensate and its momentum dependence are estimated.

1 Introduction

Since the discovery of anomalous properties [1,2] of the triangle diagram (Fig. 1) with incoming two vector and one axial-vector currents [3] many new interesting results have been gained. Recently the interest in the triangle diagram has been renewed due to the problem of an accurate calculation of higher order hadronic contributions to the muon anomalous magnetic moment via the light-by-light scattering process[1]. At low energies the dynamics of light-by-light scattering is non-perturbative, so one needs a rather realistic QCD inspired model to find a solution with the lowest model sensitivity.

The light-by-light scattering amplitude with one photon real and another photon having momenta much smaller than the other two can be analyzed using the operator product expansion (OPE) technique. In this special kinematics the amplitude is factorized into the amplitude depending on the largest photon momenta and the triangle amplitude involving the axial current A and two electromagnetic currents (one soft $\widetilde{V}$ and one virtual V). The corresponding triangle amplitude, which can be viewed as a mixing between the axial and vector currents in the external electromagnetic field, were considered recently in [5,6]. It can be expressed in terms of the two independent invariant functions, the longitudinal one $w_{\rm L}$ and the transversal one $w_{\rm T}$ with respect to the axial current index. In perturbative theory for massless quarks (the chiral limit) one has for space-like momenta q $\left(q^2 \geq 0\right)$

$$w_{\rm L}\left(q^2\right) = 2w_{\rm T}\left(q^2\right) = \frac{2}{q^2}. \qquad (1)$$

The appearance of the longitudinal structure is a consequence of the axial Adler–Bell–Jackiw anomaly [1, 2]. Because there are no perturbative (Fig. 1b) [7] and non-perturbative (Fig. 1c) [8] corrections to the axial anomaly, the invariant function $w_{\rm L}$ remains intact when the interaction with gluons is taken into account. Nonrenormalization of the longitudinal part follows from the 't Hooft consistency condition [8], i.e. the exact quark–hadron duality. In QCD this duality is realized as a correspondence between the infrared singularity of the quark triangle and the massless pion pole in terms of hadrons. It was shown in [6] (see also [9]) that in the non-singlet channel the transversal structure $w_{\rm T}$ is also free from perturbative corrections. OPE analysis indicates that at large q the leading non-perturbative power corrections to $w_{\rm T}$ can only appear starting with terms $\sim 1/q^6$ containing the matrix elements of the operators of dimension six [10]. Thus, the transversal part of the triangle with a soft momentum in one of the vector currents has no perturbative corrections; nevertheless it is modified non-perturbatively.

In the present work we analyze in the framework of the instanton liquid model [11] the non-perturbative properties of the triangle diagram in the kinematics specified above (see Sect. 2 for further details). The model is based on the representation of the QCD vacuum as an ensemble of strong vacuum fluctuations of the gluon field, instantons. They characterize the non-local properties of the QCD vacuum [12–14]. The interaction of light u, d quarks in the instanton vacuum can be described in terms of the effective 't Hooft four-quark action with nonlocal kernel induced by quark zero modes in the instanton field (Sect. 3). The gauged version of the model [15–17] meets the symmetry properties with respect to the exter-

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[1] See, e.g., [4,5] and references therein.

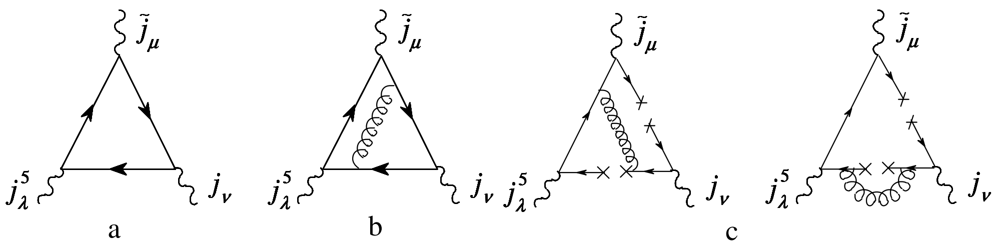

Fig. 1. Quark triangle diagram, (*a*); perturbative gluon, (*b*), and four-quark condensate, (*c*), corrections to it

nal gauge fields (Sect. 4), and the corresponding vertices satisfy the Ward–Takahashi identities. Below, in Sect. 5, we demonstrate how the anomalous structure $w_{\rm L}$ is saturated within the instanton liquid model. We also calculate the transversal invariant function $w_{\rm T}$ at arbitrary space-like q and show that within the instanton model at large q^2 there are no power corrections to this structure. The non-perturbative corrections to $w_{\rm T}$ at large q^2 have an exponentially decreasing behavior related to the short distance properties of the instanton non-locality in the QCD vacuum. We also estimate the slope of the transversal invariant function at zero virtuality.

When light quark current masses, m_f $(f=u,d)$, are switched on, additional OPE structures appear, with the leading one being of dimension four, $\sim m_f\bar{q}\,\sigma_{\alpha\beta}q$. Its matrix element between vacuum and soft photon state is proportional to the quark condensate magnetic susceptibility introduced in [18]. Using the expansion of the triangle amplitude in inverse powers of the momentum transfer squared we will derive an expression for the magnetic susceptibility in the instanton model and find its momentum dependence (Sect. 6).

2 The structure of the $VA\widetilde{V}$ correlator

We will employ a tensor decomposition of the VVA triangle graph amplitude as suggested originally by Rosenberg [3] for the general kinematics of the incoming momenta:

$$\begin{aligned}T_{\mu\nu\lambda}&(q_1,q_2)\\ &= A_1 q_1^\rho\varepsilon_{\rho\mu\nu\lambda}+A_2 q_2^\rho\varepsilon_{\rho\mu\nu\lambda}+A_3 q_1^\nu q_1^\rho q_2^\sigma\varepsilon_{\rho\sigma\mu\lambda}\\ &\quad +A_4 q_2^\nu q_1^\rho q_2^\sigma\varepsilon_{\rho\sigma\mu\lambda}+A_5 q_1^\mu q_1^\rho q_2^\sigma\varepsilon_{\rho\sigma\nu\lambda}+A_6 q_2^\mu q_1^\rho q_2^\sigma\varepsilon_{\rho\sigma\nu\lambda},\end{aligned} \tag{2}$$

where q_1 and q_2 are the vector field momenta with corresponding Lorentz indices μ and ν. The coefficients $A_j = A_j(q_1,q_2), j=1,...,6$ are the Lorentz invariant amplitudes. The vector Ward identities provide a gauge invariant definition of the A_1 and A_2 amplitudes in terms of the finite amplitudes $A_k, k=3,...,6$,

$$A_1=(q_1q_2)\,A_3+q_2^2A_4,\quad A_2=(q_1q_2)\,A_6+q_1^2A_5. \tag{3}$$

In the specific kinematics when one photon ($q_2\equiv q$) is virtual and the other one (q_1) represents the external electromagnetic field and can be regarded as a real photon with the vanishingly small momentum q_1, only two invariant functions survive in the approximation linear in small q_1 [19]. It is convenient to define longitudinal and transversal with respect to the axial current index amplitudes [6]

$$\begin{aligned}w_{\rm L}\left(q^2\right)&=4\pi^2\widetilde{A}_4\left(q^2\right),\\ w_{\rm T}\left(q^2\right)&=4\pi^2\left(\widetilde{A}_4\left(q^2\right)+\widetilde{A}_6\left(q^2\right)\right),\end{aligned} \tag{4}$$

where the tilted amplitudes are $\widetilde{A}(q^2)\equiv A\,(q_1=0,q_2=q)$. In terms of w invariant functions the $VA\widetilde{V}$ amplitude becomes

$$\begin{aligned}&\widetilde{T}_{\mu\nu\lambda}(q_1,q_2)\\ &=\frac{1}{4\pi^2}\left[-w_{\rm L}q_2^\lambda q_1^\rho q_2^\sigma\varepsilon_{\rho\mu\sigma\nu}\right.\\ &\quad\left.+w_{\rm T}\left(q_2^2q_1^\rho\varepsilon_{\rho\mu\nu\lambda}-q_2^\nu q_1^\rho q_2^\sigma\varepsilon_{\rho\mu\sigma\lambda}+q_2^\lambda q_1^\rho q_2^\sigma\varepsilon_{\rho\mu\sigma\nu}\right)\right].\end{aligned} \tag{5}$$

Both structures are transversal with respect to the vector current, $q_2^\nu\widetilde{T}_{\mu\nu\lambda}=0$. As for the axial current, the first structure is transversal with respect to q_2^λ, while the second one is longitudinal and thus anomalous.

The amplitude for the triangle diagrams (Fig. 1) can be written as a correlator of the axial current j_λ^5 and two vector currents j_ν and $\tilde{j}_\mu$:

$$T_{\mu\nu\lambda}=-\int {\rm d}^4x{\rm d}^4y\,{\rm e}^{iqx-iky}\,\langle 0|\,T\{j_\nu(x)\,\tilde{j}_\mu(y)\,j_\lambda^5(0)\}|0\rangle\,, \tag{6}$$

where for light u and d quarks one has

$$j_\mu=\bar{q}\,\gamma_\mu Qq,\quad j_\lambda^5=\bar{q}\,\gamma_\lambda\gamma_5\tau_3q\,,$$

the quark field q_f^i has color (i) and flavor (f) indices, the charge matrix is $Q=\frac{1}{2}\left(\frac{1}{3}+\tau_3\right)$, and the tilted current is for the soft momentum photon vertex.

In the local theory the one-loop result for the invariant functions $w_{\rm T}$ and $w_{\rm L}$ is[2]

$$w_{\rm L}^{1-{\rm loop}}=2\,w_{\rm T}^{1-{\rm loop}}=\frac{2N_c}{3}\int_0^1\frac{{\rm d}\alpha\,\alpha(1-\alpha)}{\alpha(1-\alpha)q^2+m_f^2}\,, \tag{7}$$

where the factor $N_c/3$ is due to the color number and electric charge. The analytical result for the triangle diagram

[2] Here and below the small effects of isospin violation is neglected, considering $m_f\equiv m_u=m_d$.

with finite quark masses has been obtained in [20] by the dispersion integral method. In the chiral limit, $m_f = 0$, one gets the result (1) (with additional factor $N_c/3$).

When non-perturbative contributions to the triangle amplitude (Fig. 1c) are taken into account it was shown in [10] by using the OPE methods that at large Euclidean q^2 the difference between the longitudinal and transversal parts, $w_{\mathrm{LT}} = w_{\mathrm{L}} - 2w_{\mathrm{T}}$, starts in the chiral limit from leading, $\sim 1/q^6$, power behavior. The power terms are expected to contribute only to the transversal function w_{T}. Below we demonstrate that within the instanton liquid model in the chiral limit all power corrections to w_{T} allowed by OPE cancel each other and only exponentially suppressed corrections remain.

3 The instanton effective quark model

To study non-perturbative effects in the triangle amplitude $\widetilde{T}_{\mu\nu\lambda}$ at low and high momenta one can use the framework of the effective field model of QCD. In the low momenta domain the effect of the non-perturbative structure of the QCD vacuum becomes dominant. Since the invention of the QCD sum rule method based on the use of the standard OPE it is common to parameterize the non-perturbative properties of the QCD vacuum by using infinite towers of vacuum expectation values of the quark–gluon operators. From this point of view the non-local properties of the QCD vacuum result from the partial resummation of the infinite series of power corrections, related to the vacuum averages of the quark–gluon operators with growing dimension and may be conventionally described in terms of the non-local vacuum condensates [12,13]. This reconstruction leads effectively to non-local modifications of the propagators and effective vertices of the quark and gluon fields at small momenta.

The adequate model describing this general picture is the instanton liquid model of the QCD vacuum describing non-perturbative non-local interactions in terms of the effective action [11]. Spontaneous breaking of the chiral symmetry and dynamical generation of the momentum-dependent quark mass are naturally explained within the instanton liquid model. The non-singlet and singlet V and A current–current correlators and the vector Adler functions, have been calculated in [17,21,22] in the framework of the effective chiral model with an instanton-like non-local quark–quark interaction [16]. In the same model the pion transition form factor normalized by the axial anomaly has been considered in [23] for arbitrary photon virtualities.

We start with the non-local chirally invariant action which describes the interaction of soft quark fields [16]

$$
\begin{aligned}
S = &\int \mathrm{d}^4x\, \overline{q}_I(x)\left[\mathrm{i}\gamma^\mu D_\mu - m_f\right] q_I(x) \qquad (8)\\
&+ \frac{1}{2}G_P \int \mathrm{d}^4X \int \prod_{n=1}^{4} \mathrm{d}^4x_n f(x_n)\\
&\times \left[\overline{Q}(X-x_1,X)\ \varGamma_P Q(X,X+x_3)\overline{Q}(X-x_2,X)\right.\\
&\left.\times \varGamma_P Q(X,X+x_4)\right],
\end{aligned}
$$

where $D_\mu = \partial_\mu - \mathrm{i}V_\mu(x) - \mathrm{i}\gamma_5 A_\mu(x)$ and the matrix product $\varGamma_P \otimes \varGamma_P = (1\otimes 1 + \mathrm{i}\gamma_5\tau^a \otimes \mathrm{i}\gamma_5\tau^a)$ provides the spin–flavor structure of the interaction. In (8) $\overline{q}_I = (\overline{u},\overline{d})$ denotes the flavor doublet field of the dynamically generated quarks, G_P is the four-quark coupling constant, and τ^a are the Pauli isospin matrices. The separable non-local kernel of the interaction determined in terms of the form factors $f(x)$ is motivated by the instanton model of the QCD vacuum.

In order to make the non-local action gauge invariant with respect to the external gauge fields $V^a_\mu(x)$ and $A^a_\mu(x)$, we define in (8) the delocalized quark field, $Q(x)$, by using the Schwinger gauge phase factor

$$
\begin{aligned}
&Q(x,y)\\
&= P\exp\left\{\mathrm{i}\int_x^y \mathrm{d}z_\mu \left[V^a_\mu(z) + \gamma_5 A^a_\mu(z)\right] T^a\right\} q_I(y),\\
&\overline{Q}(x,y) = Q^\dagger(x,y)\gamma^0, \qquad (9)
\end{aligned}
$$

where P is the operator of ordering along the integration path, with y denoting the position of the quark and x being an arbitrary reference point. The conserved vector and axial-vector currents have been derived earlier in [16,17,22].

The dressed quark propagator, $S(p)$, is defined by

$$
S^{-1}(p) = \mathrm{i}\widehat{p} - M(p^2), \qquad (10)
$$

with the momentum-dependent quark mass found as the solution of the gap equation

$$
\begin{aligned}
M(p^2) = &\ m_f \qquad (11)\\
&+ 4G_P N_f N_c f^2(p^2) \int \frac{\mathrm{d}^4k}{(2\pi)^4} f^2(k^2) \frac{M(k^2)}{k^2 + M^2(k^2)}.
\end{aligned}
$$

The formal solution is expressed as [15]

$$
M(p^2) = m_f + (M_q - m_f) f^2(p^2), \qquad (12)
$$

with constant $M_q \equiv M(0)$ determined dynamically from (11) and the momentum-dependent $f(p)$ is the normalized four-dimensional Fourier transform of $f(x)$ given in the coordinate representation.

The non-local function $f(p)$ describes the momentum distribution of the quarks in the non-perturbative vacuum. Given the non-locality of $f(p)$, the light quark condensate in the chiral limit, $M(p) = M_q f^2(p)$, is expressed as

$$
\langle 0\left|\overline{q}q\right|0\rangle = -N_c \int \frac{\mathrm{d}^4p}{4\pi^4} \frac{M(p^2)}{p^2 + M^2(p^2)}. \qquad (13)
$$

Its n-moment is proportional to the vacuum expectation value of the quark condensate with covariant with respect to the gluon field derivative squared D^2 to the nth power

$$
\langle 0\left|\overline{q}D^{2n}q\right|0\rangle = -N_c \int \frac{\mathrm{d}^4p}{4\pi^4} p^{2n} \frac{M(p^2)}{p^2 + M^2(p^2)}, \qquad (14)
$$

The nth moment of the quark condensate appears as a coefficient of the Taylor expansion of the non-local quark condensate defined by [12]

$$C(x) = \left\langle 0 \left| \overline{q}(0) P \exp \left[\mathrm{i} \int_0^x A_\mu(z) \, \mathrm{d}z_\mu \right] q(x) \right| 0 \right\rangle, \quad (15)$$

with the gluon Schwinger phase factor inserted for gauge invariance, and the integral is over the straight line path. The smoothness of $C(x)$ near $x^2 = 0$ leads to the existence of the quark condensate moments in the LHS of (14) for any n. In order to make the integral in the RHS of (14) convergent the non-local function $f(p)$ for large arguments must decrease faster than any inverse power of p^2, e.g., like some exponential[3]:

$$f(p) \sim \exp(-\mathrm{const}{\cdot}p^\alpha), \ \alpha > 0 \quad \text{as} \quad p^2 \to \infty. \quad (16)$$

Note that the operators entering the matrix elements in (14) and (15) are constructed from the QCD quark and gluon fields. The RHS of (14) is the value of the matrix elements of QCD defined operators calculated within the effective instanton model with dynamical quark fields. Within the instanton model the zero mode function $f(p)$ depends on the gauge. It is implied [13,14] that the RHS of (14) corresponds to calculations in the axial gauge for the quark effective field. It is selected among other gauges because in this gauge the covariant derivatives become the ordinary ones: $D \to \partial$, and the exponential in (15) with a straight line path is reduced to the unit. In particular it means that one uses the quark zero modes in the instanton field given in the axial gauge when defining the gauge dependent dynamical quark mass. The axial gauge at large momenta has exponentially decreasing behavior and all moments of the quark condensate exist. In principle, to calculate the gauge invariant matrix element corresponding to the of LHS of (14) it is possible to use the expression for the dynamical mass given in any gauge, but in that case the factor p^{2n} will be modified by a more complicated weight function providing invariance of the answer[4].

Furthermore, the large distance asymptotics of the instanton solution is also modified by screening effects due to the interaction of the instanton field with surrounding physical vacuum [24,25]. To take into account these effects and make the numerics simpler we shell use for the non-local function the Gaussian form

$$f(p) = \exp\left(-p^2/\Lambda^2\right), \quad (17)$$

where the parameter Λ characterizes the non-locality size of the gluon vacuum fluctuations, and it is proportional to the inverse average size of the instanton in the QCD vacuum.

The important property of the dynamical mass (11) is that at low virtualities its value is close to the constituent mass, while at large virtualities it goes to the current mass value. As we will see in Sect. 5 this property is crucial in obtaining the anomaly at large momentum transfer. The instanton liquid model can be viewed as an approximation of large-N_c QCD where the only new interaction terms, retained after integration of the high frequency modes of the quark and gluon fields down to the non-locality scale Λ at which spontaneous chiral symmetry breaking occurs, are those which can be cast in the form of four-fermion operators (8). The parameters of the model are then the non-locality scale Λ and the four-fermion coupling constant G_P.

4 Conserved vector and axial-vector currents

The quark–antiquark scattering matrix (Fig. 2) in the pseudoscalar channel is found from the Bethe–Salpeter equation:

$$\widehat{T}_P(q^2) = \frac{G_P}{1 - G_P J_{PP}(q^2)}, \quad (18)$$

with the polarization operator being

$$J_{PP}(q^2) = \int \frac{\mathrm{d}^4k}{(2\pi)^4} f^2(k) f^2(k+q) \operatorname{Tr}\left[S(k)\gamma_5 S(k+q)\gamma_5\right]. \quad (19)$$

The position of the pion state is determined as the pole of the scattering matrix

$$\det(1 - G_P J_{PP}(q^2))\big|_{q^2=-m_\pi^2} = 0. \quad (20)$$

The quark–pion vertex found from the residue of the scattering matrix is ($k' = k + q$)

$$\Gamma_\pi^a(k, k') = g_{\pi qq} \mathrm{i}\gamma_5 f(k) f(k') \tau^a,$$

with the quark–pion coupling found from

$$g_{\pi q}^{-2} = -\left.\frac{\mathrm{d}J_{PP}(q^2)}{\mathrm{d}q^2}\right|_{q^2=-m_\pi^2}, \quad (21)$$

where m_π is the physical mass of the π meson. The quark–pion coupling, $g_{\pi q}$, and the pion decay constant, f_π, are connected by the Goldberger–Treiman relation,

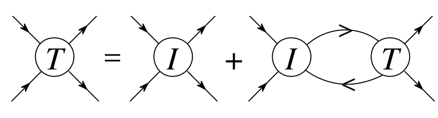

Fig. 2. Diagrammatic representation of the Bethe–Salpeter equation for the quark–quark scattering matrix, T, with non-local instanton kernel, I

[3] Very similar arguments lead the author of [30] to the conclusion that the finiteness of all transverse momenta moments of the quark distributions guarantees the exponential fall-off of the cross sections.

[4] If one would naively use the dynamical quark mass corresponding to the popular singular gauge then one finds the problem with convergence of the integrals in (14), because in this gauge there is only power-like asymptotics of $M(p) \sim p^{-6}$ at large p^2.

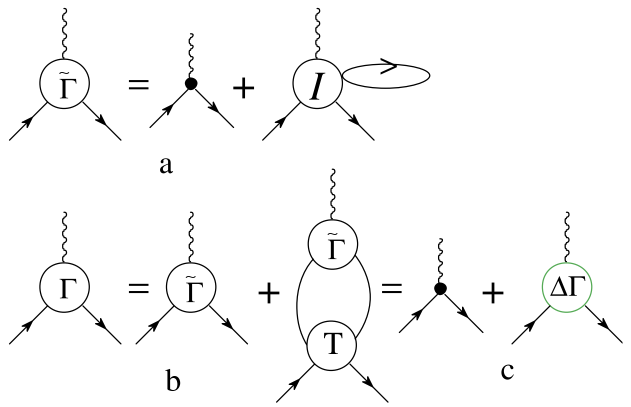

Fig. 3. Diagrammatic representation of the bare **a** and full **b** quark–current vertices. Diagram **c** shows the separation of local (fat dot) and non-local parts of the full vertex

$g_\pi = M_q/f_\pi$, which is verified to be valid in the non-local model [15], as requested by chiral symmetry.

The vector vertex following from the model (8) is (Fig. 3a)

$$\Gamma_\mu(k,k') = \gamma_\mu + (k+k')_\mu M^{(1)}(k,k'), \tag{22}$$

where $M^{(1)}(k,k')$ is the finite-difference derivative of the dynamical quark mass, q is the momentum corresponding to the current, and k (k') is the incoming (outgoing) momentum of the quark, $k' = k+q$. The finite-difference derivative of an arbitrary function F is defined as

$$F^{(1)}(k,k') = \frac{F(k') - F(k)}{k'^2 - k^2}. \tag{23}$$

The full axial vertex corresponding to the conserved axial-vector current is obtained after resummation of the quark-loop chain that results in the appearance of a term proportional to the pion propagator [16] (Fig. 3b)

$$\begin{aligned}\Gamma^5_\mu(k,k') &= \gamma_\mu\gamma_5 \\ &+ 2\gamma_5 \frac{q_\mu}{q^2} f(k) f(k') \left[J_{AP}(0) - \frac{m_f G_P J_P\left(q^2\right)}{1 - G_P J_{PP}\left(q^2\right)} \right] \\ &+ (k+k')_\mu J_{AP}(0) \frac{\left(f(k') - f(k)\right)^2}{k'^2 - k^2},\end{aligned} \tag{24}$$

where we have introduced the notation

$$\begin{aligned} J_P(q^2) &\\ &= \int \frac{\mathrm{d}^4k}{(2\pi)^4} f(k) f(k+q) \,\mathrm{Tr}\left[S(k)\gamma_5 S(k+q)\gamma_5\right], \end{aligned} \tag{25}$$

$$\begin{aligned} J_{AP}(q^2) &\\ &= 4N_c N_f \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{M(l)}{D(l)} \sqrt{M(l+q)M(l)}. \end{aligned} \tag{26}$$

The axial-vector vertex has a pole at

$$q^2 = -m_\pi^2 = m_c \langle \overline{q} q\rangle / f_\pi^2 \tag{27}$$

where the Goldberger–Treiman relation and the definition of the quark condensate have been used. The pole is related to the denominator $1 - G_P J_{PP}\left(q^2\right)$ in (24), while q^2 in the denominator is compensated by the zero from the square brackets in the limit $q^2 \to 0$. This compensation follows from the expansion of the $J(q^2)$ functions near zero:

$$J_{PP}(q^2) = G_P^{-1} + m_c \langle\overline{q}q\rangle M_q^{-2} - q^2 g_{\pi q}^{-2} + O\left(q^4\right),$$
$$J_{AP}(q^2=0) = M_q, \qquad J_P(q^2=0) = \langle\overline{q}q\rangle M_q^{-1}.$$

In the chiral limit $m_f = 0$ the second structure in square brackets in (24) disappears and the pole moves to zero.

The parameters of the model are fixed in a way typical for effective low-energy quark models. One usually fits the pion decay constant, f_π, to its experimental value, which in the chiral limit reduces to 86 MeV [26]. In the instanton model the constant, f_π, is expressed as

$$f_\pi^2 = \frac{N_c}{4\pi^2} \int_0^\infty \mathrm{d}u\, u \frac{M^2(u) - uM(u)M'(u) + u^2 M'(u)^2}{D^2(u)}, \tag{28}$$

where here and below $u = k^2$, and primes mean derivatives with respect to u: $M'(u) = \mathrm{d}M(u)/\mathrm{d}u$, etc., and

$$D\left(k^2\right) = k^2 + M^2(k).$$

One gets the values of the model parameters [22]

$$M_q = 0.24\,\mathrm{GeV}, \quad \Lambda_P = 1.11\,\mathrm{GeV}, \quad G_P = 27.4\,\mathrm{GeV}^{-2}. \tag{29}$$

5 The $VA\widetilde{V}$ correlator within the instanton liquid model

Our goal is to obtain the non-diagonal correlator of the vector current and the non-singlet axial-vector current in the external electromagnetic field ($VA\widetilde{V}$) by using the effective instanton-like model (8). In this model the $VA\widetilde{V}$ correlator is defined by (Fig. 4a)

$$\begin{aligned}\widetilde{T}_{\mu\nu\lambda}(q_1,q_2) &\\ &= -2N_c \int \frac{\mathrm{d}^4k}{(2\pi)^4} \mathrm{Tr}\left[\Gamma_\mu(k+q_1,k) S(k+q_1)\right. \\ &\quad \times \Gamma^5_\lambda(k+q_1,k-q_2) S(k-q_2) \\ &\quad \left.\times\, \Gamma_\nu(k,k-q_2) S(k)\right],\end{aligned} \tag{30}$$

where the quark propagator, the vector and the axial-vector vertices are given by (10), (22) and (24), respectively. The structure of the vector vertices guarantees that the amplitude is transversal with respect to the vector indices

$$\widetilde{T}_{\mu\nu\lambda}(q_1,q_2) q_1^\mu = \widetilde{T}_{\mu\nu\lambda}(q_1,q_2) q_2^\nu = 0, \tag{31}$$

and the Lorentz structure of the amplitude is given by (5).

It is convenient to express (30) as a sum of the contribution where all vertices are local (Fig. 4b), and the

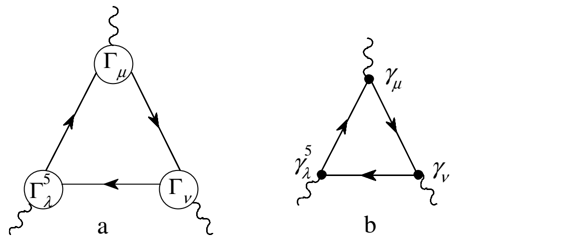

Fig. 4. Diagrammatic representation of the triangle diagram in the instanton model with dressed quark lines and full quark–current vertices (a); and part of the diagram when all vertices are local one (b)

rest contribution containing non-local parts of the vertices (Fig. 3c). Further results in this section will concern the chiral limit.

The contributions of diagram 4b to the invariant functions at space-like momentum transfer, $q^2 \equiv q_2^2$, are given by

$$\widetilde{A}_4^{(\mathrm{L})}\left(q^2\right) = \frac{N_c}{9q^2}\int \frac{\mathrm{d}^4k}{\pi^4}\frac{1}{D_+^2 D_-}\left[k^2 - 4\frac{(kq)^2}{q^2} + 3\,(kq)\right], \tag{32}$$

$$\widetilde{A}_6^{(\mathrm{L})}\left(q^2\right) = -\frac{1}{2}\widetilde{A}_4^{(\mathrm{L})}\left(q^2\right), \tag{33}$$

where the notation used here and below is

$$k_+ = k, \quad k_- = k - q, \quad k_\perp^2 = k_+k_- - \frac{(k_+q)\,(k_-q)}{q^2},$$
$$D_\pm = D(k_\pm^2), \quad M_\pm = M(k_\pm^2), \quad f_\pm = f(k_\pm^2).$$

At large q^2 one has the expansion

$$\widetilde{A}_4^{(\mathrm{L})}\left(q^2 \to \infty\right) = \frac{N_c}{6\pi^2}\left(\frac{1}{q^2} + \frac{a_{(4)}^{(\mathrm{L})}}{q^4} + \frac{a_{(6)}^{(L)}}{q^6} + O\left(q^{-8}\right)\right), \tag{34}$$

with coefficients given by

$$a_{(4)}^{(\mathrm{L})} = -\int_0^\infty \mathrm{d}u \frac{M^2(u)}{D^2(u)}\left(2u + M^2(u)\right), \tag{35}$$

$$a_{(6)}^{(\mathrm{L})} = -\frac{2}{3}\int_0^\infty \mathrm{d}u \frac{uM^2(u)\left(u + 2M^2(u)\right)}{D^2(u)}. \tag{36}$$

It is clear that the contribution (32) saturates the anomaly at large q^2. The reason is that the leading asymptotics of (32) is given by the configuration where the large momentum is passing through all quark lines. Then the dynamical quark mass $M(k)$ reduces to zero and the asymptotic limit of the triangle diagram with dynamical quarks and local vertices coincides with the standard triangle amplitude with massless quarks and, thus, it is independent of the model.

The contribution to the form factors when the non-local parts of the vector and axial-vector vertices are taken into account is given by

$$\widetilde{A}_4^{(\mathrm{NL})}\left(q^2\right) = \frac{N_c}{3q^2}\int \frac{\mathrm{d}^4k}{\pi^4}\frac{1}{D_+^2 D_-}\left\{M_+\left[M_+ - \frac{4}{3}M_+'k_\perp^2\right] - M^{2(1)}(k_+,k_-)\left(2\frac{(kq)^2}{q^2} - (kq)\right)\right\}. \tag{37}$$

One has for the leading terms of the large q^2 asymptotics

$$\widetilde{A}_4^{(\mathrm{NL})}\left(q^2 \to \infty\right) = \frac{N_c}{6\pi^2}\left(\frac{a_{(4)}^{(NL)}}{q^4} + \frac{a_{(6)}^{(\mathrm{NL})}}{q^6} + O\left(q^{-8}\right)\right), \tag{38}$$

with coefficients given by

$$a_{(4)}^{(\mathrm{NL})} = 2\int_0^\infty \mathrm{d}u \frac{uM(u)}{D^2(u)}\left(M(u) - uM'(u)\right), \tag{39}$$
$$a_{(6)}^{(\mathrm{NL})} = \frac{2}{3}\int_0^\infty \mathrm{d}u \frac{u^3M(u)\,M'(u)}{D^2(u)}.$$

In the sum of the two contributions both power corrections with coefficients $a_{(4)}$ and $a_{(6)}$ are canceled. To prove cancellation of the coefficients one needs to use integration by parts.

Summing analytically the local (32) and non-local (37) parts provides us with the result required by the axial anomaly:

$$w_{\mathrm{L}}(q^2) = 4\pi^2\widetilde{A}_4\left(q^2\right) = \frac{2N_c}{3}\frac{1}{q^2}. \tag{40}$$

Figure 5 illustrates how different contributions saturate the anomaly. Note that at zero virtuality the saturation of anomaly follows from the anomalous diagram of pion decay in two photons. This part is due to the triangle diagram involving the non-local part of the axial vertex and the local parts of the photon vertices. The result (40) is in agreement with the statement about the absence of non-perturbative corrections to the longitudinal invariant function following from the 't Hooft duality arguments. Earlier this consistency has also been demonstrated within the QCD sum rules [27,28] and within the dispersion method [29] considering the lowest orders of the expansion of the triangle diagram in vacuum condensates.

For $\widetilde{A}_6^{(\mathrm{NL})}\left(q^2\right)$ invariant function one gets the result

$$\begin{aligned}\widetilde{A}_6^{(\mathrm{NL})}\left(q^2\right) &= -\frac{N_c}{6q^2}\int \frac{\mathrm{d}^4k}{\pi^4}\frac{1}{D_+^2 D_-}\left\{(M_+ + M_-)\right.\\ &\times\left[M_+ - \frac{(kq)}{q^2}(M_+ - M_-)\right]\\ &-\frac{2}{3}M_+'\left[2k_\perp^2 M_+\right.\\ &\left.\left.-M_-\frac{q^2}{k_+^2 - k_-^2}\left(k^2 + 2\frac{(kq)^2}{q^2} - 6\,(kq)\frac{k^2}{q^2}\right)\right]\right\}\\ &+\frac{2N_c}{9q^2}\int \frac{\mathrm{d}^4k}{\pi^4}\frac{\sqrt{M_+M_-}}{D_+^2 D_-}\end{aligned} \tag{41}$$

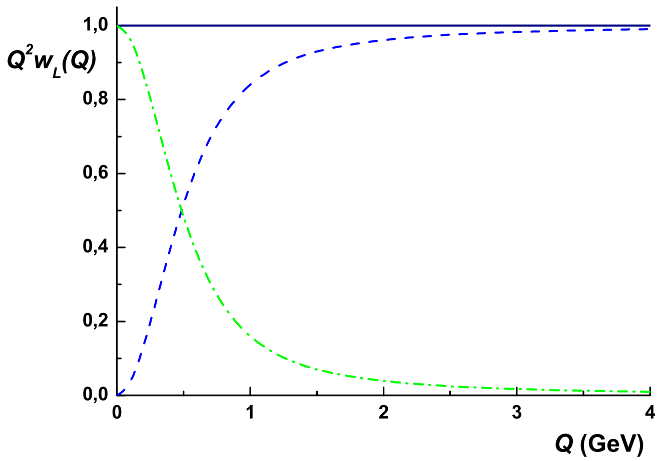

Fig. 5. The normalized w_{L} invariant function constrained by the ABJ anomaly from the triangle diagram Fig. 4a (solid line) and different contributions to it: from the local part, Fig. 4b, (dashed line), and from the non-local part (dash-dotted line)

$$\times \frac{k_\perp^2}{k_+^2 - k_-^2}\left[M_+ - M_- - 2M_+'(kq)\right].$$

Then, let us consider the combination of invariant functions which show non-perturbative dynamics:

$$\begin{aligned} w_{\mathrm{LT}}(q^2) &\equiv w_{\mathrm{L}}(q^2) - 2w_{\mathrm{T}}(q^2) \\ &= -4\pi^2\left[\widetilde{A}_4\left(q^2\right) + 2\widetilde{A}_6\left(q^2\right)\right]. \end{aligned} \tag{42}$$

From (33) we see that the contribution to w_{LT} from the triangle diagram 4b with local vertices is absent. In the sum of $\widetilde{A}_4\left(q^2\right)$ and $\widetilde{A}_6\left(q^2\right)$ a number of cancellations take place and the final result is quite simple:

$$\begin{aligned} &w_{\mathrm{LT}}\left(q^2\right) \\ &= \frac{4N_c}{3q^2}\int\frac{\mathrm{d}^4k}{\pi^2} \\ &\times\frac{\sqrt{M_-}}{D_+^2 D_-}\left\{\sqrt{M_-}\left[M_+ - \frac{2}{3}M_+'\left(k^2 + 2\frac{(kq)^2}{q^2}\right)\right]\right. \\ &\quad -\frac{4}{3}k_\perp^2\left[\sqrt{M_+}M^{(1)}(k_+,k_-)\right. \\ &\qquad \left.\left. -\ 2\,(kq)\,M_+'\sqrt{M}^{(1)}(k_+,k_-)\right]\right\}. \end{aligned} \tag{43}$$

The behavior of $w_{\mathrm{LT}}(q^2)$ is presented in Fig. 6. In the above expression the integrand is proportional to the product of the non-local form factors $f\left(k_+^2\right) f\left(k_-^2\right)$ depending on the quark momenta passing through different quark lines. Then it becomes evident that the large q^2 asymptotics of the integral is governed by the asymptotics of the non-local form factor $f\left(q^2\right)$ which is exponentially suppressed (16). Thus, within the instanton model the distinction between the longitudinal and transversal parts is exponentially suppressed at large q^2 and all power corrections allowed by OPE cancel each other. Recently, it was

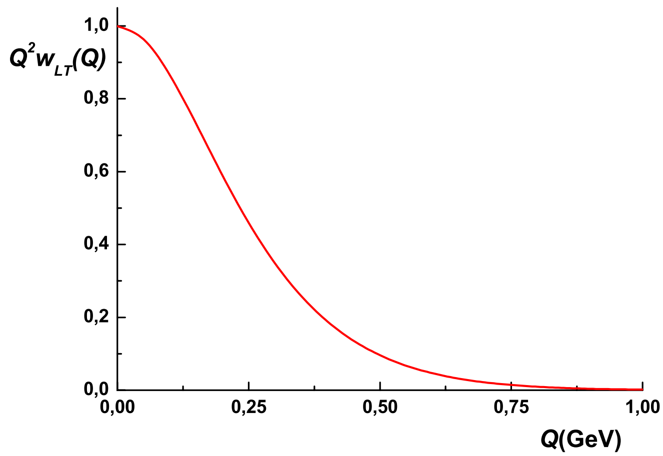

Fig. 6. The normalized w_{LT} invariant function versus Q predicted by the instanton model from the diagram Fig. 4a

proven that the relation

$$w_{\mathrm{LT}}\left(q^2\right) = 0, \tag{44}$$

which holds in the chiral limit at the one-loop level, gets no perturbative corrections from gluon exchanges [6]. The instanton liquid model indicates that it may be possible that due to the anomaly this relation is violated at large q^2 only exponentially.

We also find numerical values of the slope of the invariant function w_{LT} at zero virtuality:

$$\left.\frac{\partial q^2 w_{\mathrm{LT}}(q^2)}{\partial q^2}\right|_{q^2=0}(\mu_{\mathrm{Inst}}) = -3.8\,\mathrm{GeV}^{-2}, \tag{45}$$

and its width

$$\lambda_{\mathrm{LT}}^2 \equiv \int u^2 w_{\mathrm{LT}}(u)\,\mathrm{d}u\cdot\left(\int u w_{\mathrm{LT}}(u)\,\mathrm{d}u\right)^{-1} = 0.54\,\mathrm{GeV}^2. \tag{46}$$

6 Magnetic susceptibility of the quark condensate

In this section we consider the leading power corrections to w_{L} and w_{T} resulting from the inclusion into the consideration of the current quark mass, m_f. The appearance of this kind of power corrections is already clear from the perturbative expression (7). In OPE the leading, by dimension, correction to the invariant functions $w_{\mathrm{L,T}}(q^2)$ is

$$\Delta w_{\mathrm{L}} = 2\,\Delta w_{\mathrm{T}} = \frac{4m_f\kappa_f}{3q^4}, \tag{47}$$

where κ_f are the matrix element of dimension 3 operators:

$$\mathcal{O}_f^{\alpha\beta} = -\mathrm{i}\,\bar{q}_f\,\sigma^{\alpha\beta}\gamma_5\,q^f \tag{48}$$

between the soft photon and vacuum states. The proportionality to m_f in (47) is in correspondence with chirality arguments.

In perturbation theory the matrix element κ_f of the chirality-flip operator O_f is proportional to m_f. Non-perturbatively, however, due to spontaneous breaking of the chiral symmetry κ_f does not vanish at $m_f = 0$. It is convenient to introduce the magnetic susceptibility χ_{m} normalized by the quark condensate [18]

$$\kappa_f = -4\pi^2 \langle \bar{q}q \rangle \chi_{\mathrm{m}} .$$

Considering OPE this representation emphasizes that the magnetic susceptibility for the non-diagonal vector–axial-vector correlator in the external electromagnetic field plays a role similar to the quark condensate for the diagonal correlators of vector and axial-vector currents.

In the instanton model the $VA\widetilde{V}$ correlator is given by (30) with the quark propagator, the vector and the axial-vector vertices defined by (10), (22) and (24), with the current quark mass, m_f, being included. Keeping in the calculation only terms linear in the current quark mass one finds at large q^2 the correction at twist 4 level (47) for the contribution of diagram Fig. 4b:

$$\Delta\widetilde{A}_4^{(\mathrm{L})}\left(q^2 \to \infty\right) = -\frac{1}{q^4}\frac{2m_f}{3}\frac{N_c}{\pi^2}\int \mathrm{d}u \frac{u^2 M(u)}{D^3(u)}, \quad (49)$$

and from the non-local part

$$\begin{aligned}&\Delta\widetilde{A}_4^{(\mathrm{NL})}\left(q^2 \to \infty\right) \\ &= -\frac{1}{q^4}\frac{m_f}{3}\frac{N_c}{\pi^2}\int \mathrm{d}u \frac{uM(u)}{D^3(u)} \\ &\quad \times \left[-u + 3M^2(u) - 4uM(u)M'(u)\right]. \quad (50)\end{aligned}$$

The leading asymptotics linear in the current mass for the invariant function $\widetilde{A}_6$ is given by the relation

$$\Delta\widetilde{A}_6\left(q^2 \to \infty\right) = -\frac{1}{2}\Delta\widetilde{A}_4\left(q^2 \to \infty\right), \quad (51)$$

which is in accordance with OPE (47).

Then, summing up the contributions (49) and (50) and comparing the result at large q^2 with OPE one gets the magnetic susceptibility in the form

$$\chi_{\mathrm{m}}(\mu_{\mathrm{Inst}}) = -\frac{1}{\langle 0|\bar{q}q|0\rangle}\frac{N_c}{4\pi^2}\int \mathrm{d}u \frac{u\left(M(u) - uM'(u)\right)}{D^2(u)}, \quad (52)$$

where the quark condensate is defined in (13).

Alternatively, to get (52) we may simply calculate the matrix element of $\mathcal{O}_f^{\alpha\beta}$ (48) between the vacuum and one real photon state and use (22) for the quark–photon vertex. In this way it is easy to show that the result (52) stays unchanged when one includes the vector meson degrees of freedom. Indeed, in the extended model [22,31] the vector vertex gets a contribution from the vector ρ and ω mesons in the form

$$\begin{aligned}&\Delta\Gamma_\mu^a(p,p') \quad (53)\\ &= \left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\gamma_\nu T^a \frac{G_V f^V(p) f^V(p')}{1 - G_V J_V^T(q^2)} B_V\left(q^2\right),\end{aligned}$$

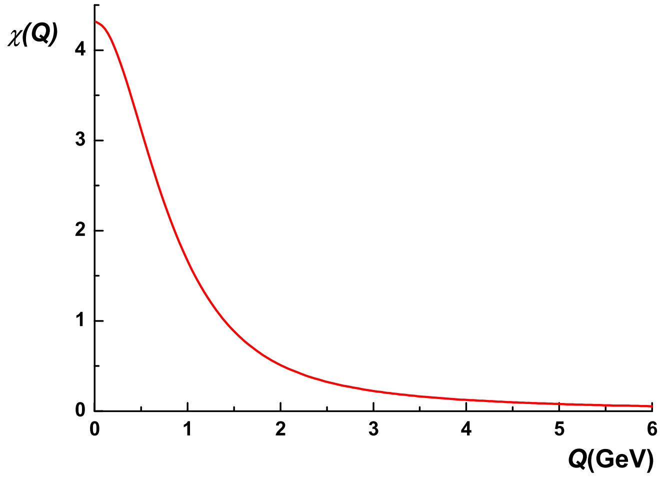

Fig. 7. The momentum dependence of the magnetic susceptibility of the quark condensate

where T^a is a flavor matrix, $f^V(p)$, $J_V^T\left(q^2\right)$, G_V are the non-local form factors, the polarization operator and four-quark coupling in the vector channel, respectively. Due to conservation of the vector current one has $B_V\left(q^2 = 0\right) = 0$ and thus there is no contribution to the magnetic susceptibility.

It is easy also to derive the momentum dependence of the magnetic susceptibility

$$\begin{aligned}\chi_{\mathrm{m}}\left(q^2\right) &= -\frac{N_c}{\langle 0|\bar{q}q|0\rangle} \\ &\times \int \frac{\mathrm{d}^4k}{4\pi^4}\frac{1}{D_+D_-}\left\{\left[M_+ - \frac{kq}{q^2}(M_+ - M_-)\right]\right. \quad (54)\\ &\left.\times\left(1 + B_V\left(q^2\right) f_+^V f_-^V\right) - \frac{2}{3}k_\perp^2 M^{(1)}(k_+,k_-)\right\},\end{aligned}$$

presented in Fig. 7. At large q the integral in (54) is proportional to the quark condensate providing the correct asymptotic result

$$\chi_{\mathrm{m}}(q \to \infty) = \frac{2}{q^2}. \quad (55)$$

Recently, (52) has been obtained in a more complicated way in [32]. Also note that the instanton model does not support the use of the pion dominance for an estimate of the magnetic susceptibility as it was attempted in [6]. The reason is that the pion pole in the axial vertex (24) is accompanied by the exponentially suppressed residue $J_P\left(q^2\right)$. Thus, it does not contribute to the twist 4 coefficient.

Given the model parameters (29) one finds the numerical values for the quark condensate and the magnetic susceptibility

$$\begin{aligned}\langle 0|\bar{q}q|0\rangle(\mu_{\mathrm{Inst}}) &= -(214\,\mathrm{GeV})^3, \\ \chi_{\mathrm{m}}(\mu_{\mathrm{Inst}}) &= 4.32\,\mathrm{GeV}^{-2}, \quad (56)\end{aligned}$$

where μ_{Inst} is the normalization scale typical for instanton fluctuations. To leading-logarithmic accuracy the scale dependence of these values is predicted by QCD to be

$$\langle 0 | \overline{q}q | 0 \rangle (\mu) = L^{-\gamma_{\overline{q}q}/b} \langle 0 | \overline{q}q | 0 \rangle (\mu_0) ,$$
$$\chi_{\rm m} (\mu) = L^{-(\gamma_0 - \gamma_{\overline{q}q})/b} \chi_{\rm m} (\mu_0) , \qquad (57)$$

where $L = \alpha_{\rm s}(\mu) / \alpha_{\rm s}(\mu_0)$, $b = (11N_c - 2n_f)/3$, $\gamma_{\overline{q}q} = -3C_F$ is the anomalous dimension of the quark condensate, and $\gamma_0 = C_F$ is the anomalous dimension of the chiral-odd local operator of leading twist, $C_F = 4/3$. We may fix the normalization scale of the model by comparing the value of the condensate with that found in the QCD sum rule at some standard normalization point: $\langle 0 | \overline{q}q | 0 \rangle (\mu_0 = 1\,{\rm GeV}) = -(240\,{\rm GeV})^3$. Then one finds $L = 2.17$ which corresponds to the normalization point $\mu_{\rm Inst} \approx 0.5\,{\rm GeV}$, with the QCD constant for three flavors being $\Lambda_{\rm QCD}^{(n_f=3)} = 296\,{\rm MeV}$. The rescaled magnetic susceptibility calculated in the model will be

$$\chi_{\rm m} (\mu_{\rm Inst} = 1\,{\rm GeV}) = 2.73\,{\rm GeV}^{-2}, \qquad (58)$$

which is in rather good agreement with the latest numerical value of $\chi_{\rm m}$ obtained with the QCD sum rule fit [33]: $\chi_{\rm m} (\mu_{\rm SR} = 1\,{\rm GeV}) = (3.15 \pm 0.3)\ {\rm GeV}^{-2}$. The phenomenology of hard exclusive processes sensitive to the magnetic susceptibility $\chi_{\rm m}$ [33] will possibly help to fix its value.

7 Conclusions

In the framework of the instanton liquid model we calculated for arbitrary space-like momenta transfer the non-diagonal correlator of the vector and non-singlet axial-vector currents in the background of a soft vector field. In this case we find that at large momenta the non-perturbative power corrections are absent in the chiral limit for the transversal part, $w_{\rm T}$, of the triangle diagram. The transversal part is corrected only by exponentially small terms which reflects the non-local structure of the QCD vacuum. Within the instanton model the saturation of the anomalous, longitudinal $w_{\rm L}$ structure is demonstrated explicitly. The non-diagonal correlator with singlet axial-vector current characterized by the presence of the gluonic $U_A(1)$ anomaly has been considered in [34]. Using the instanton liquid model we also derived an expression for the quark condensate magnetic susceptibility and its momentum dependence.

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